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Domain wall and de Sitter solutions of gauged supergravity

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Abstract: BPS domain wall solutions of gauged supergravities are found, including those theories which have non-compact gauge groups. These include models that have both an unstable de Sitter solution and stable domain wall solutions.

Keywords: M-Theory, p-branes, Superstring Vacua.

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Contents

1. Introduction

Scalar potentials whose dependence on one of the scalars fields ϕ is of the form

$$V = \Lambda - \alpha(1 - \cosh(a\phi)) \tag{1.1}$$

occur in many supergravity theories, with extrema of V giving anti-de Sitter or de Sitter solutions with cosmological constant Λ. There are many examples with $\Lambda < 0$, such as the $D = 4$, $N = 4$ gauged supergravity of [1], the $D = 4$, $N = 8$ gauged supergravity with gauge group SO(8) [2] and the $D = 5$, $N = 8$ gauged supergravity with gauge group SO(6) [3, 4]. The case $\Lambda > 0$ also occurs, for example in the $D = 4$, $N = 4$ gauged supergravity of [5], the $D = 4$, $N = 8$ gauged supergravity with gauge group $\mathrm{SO}(4,4)$ [6, 7] and the $D = 5$, $N = 8$ gauged supergravity with gauge group $\mathrm{SO}(3,3)$ [3]. These solutions were lifted to solutions of supergravity in 10 or 11 dimensions in [8]; see [9] for further discussion. The supergravities with $\Lambda < 0$ typically have stable maximally supersymmetric AdS vacua, but the de Sitter solutions arising when $\Lambda > 0$ necessarily break all supersymmetries, and moreover are unstable as the potential is unbounded below. This raises the question as to whether these theories have stable vacua. It will be shown here that such $\Lambda > 0$

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supergravities in D dimensions do have BPS domain wall solutions, i.e. solutions with $D-1$ dimensional Poincaré invariance and which preserve half the supersymmetries, so that these solutions are stable vacua for such theories.

More generally, there is a large class of gauged supergravity theories, many of which do not have AdS or Minkowski space solutions. However, it will be seen that they do have supersymmetric domain wall solutions, which are candidate ground-states. They typically also admit supersymmetric electrovac or magnetovac solutions, which are product space solutions with an electric or magnetic flux on one of the two factors [10].

There is then a class of supersymmetric models in $D = 5$ which have both a $D = 5$ de Sitter solution and a BPS domain wall solution; these include the $\mathrm{SO}(3,3)$ gauged $N = 8$ theory, but there are also models with less supersymmetry and more adjustable parameters. If there are such models in which the BPS solution admits a brane-world interpretation giving effective 4-dimensional physics as in [11], there is the interesting possibility of cosmological models which could have a phase of 5-dimensional inflation followed by a transition to a brane-world scenario with effective four-dimensional physics. The models that will be discussed here do not fit in with such an interpretation, but the possibility of finding models with bulk inflation followed by a transition to a brane-world phase seems worth pursuing.

The model that will be analysed here is D-dimensional gravity coupled to a single scalar ϕ, with lagrangian

$$\mathcal{L} = \sqrt{-\det g}\left[\frac{1}{2}R - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right]. \tag{1.2}$$

Of particular interest will be truncations of supergravities down to such models, with the truncations chosen such that critical points of $V(\phi)$ are also critical points of the full supergravity potential, by an argument of Warner [12]. Following [13], the potential will be taken to be of the form

$$V = 2(D-2)\left[(D-2)(w')^2 - (D-1)w^2\right] \tag{1.3}$$

for some 'superpotential' $w(\phi)$. The domain-wall ansatz for the metric is

$$ds^2 = e^{2A(r)}ds^2\left(\mathbb{E}^{(1,D-2)}\right) + dr^2 \tag{1.4}$$

with scalar field $\phi(r)$ depending only on the transverse coordinate r. Such solutions can be interpreted as representing renormalization group flows, with monotonic C-function [14, 15, 13]

$$\mathcal{C} = \frac{\mathcal{C}_0}{[\partial_r A(r)]^{D-2}}. \tag{1.5}$$

Critical points of the potential V correspond to RG fixed points, but there are domain wall solutions even for potentials V without critical points.

The domain wall solutions of [13] are such configurations satisfying the following pair of first-order equations:

$$\begin{aligned}\partial_r A &= \mp 2w(\phi) \\ \partial_r \phi &= \pm 2(D-2)w'(\phi)\end{aligned} \tag{1.6}$$

with one or other choice of sign. The second-order equations following from (1.2) are then satisfied. These equations were found by seeking solutions which extremise the energy

$$E[A,\phi] = \frac{1}{2}\int_{-\infty}^{\infty} dr\, e^{(D-1)A}\left[(\partial_r\phi)^2 - (D-1)(D-2)(\partial_r A)^2 + 2V\right]. \tag{1.7}$$

The equations (1.6) also follow from demanding the existence of spinors satisfying

$$(D_m + w\Gamma_m)\epsilon = 0 \tag{1.8}$$

and

$$\left[\Gamma^m\partial_m\phi - 2(D-2)w'\right]\epsilon = 0\,. \tag{1.9}$$

For solutions satisfying (1.6), the spinors satisfying (1.8), (1.9) are

$$\epsilon = e^{A/2}\epsilon_0 \tag{1.10}$$

with ϵ_0 a constant spinor satisfying

$$\Gamma_r\epsilon_0 = \pm\epsilon_0\,. \tag{1.11}$$

For a supergravity theory in a background with the only non-vanishing fields being the metric and a single scalar ϕ, there will be a supersymmetry of the background for each spinor ϵ satisfying the Killing spinor conditions. The condition from the vanishing of the gravitino variation is typically of the form

$$D_m\epsilon^a + \Gamma_m W^a{}_b\epsilon^b = 0\,, \tag{1.12}$$

where $a = 1, 2, \ldots, N$ labels the supersymmetries and $W^a{}_b(\phi)$ is a scalar-dependent matrix. If one of its eigenvalues is $w(\phi)$ with multiplicity m, then there are Killing spinors corresponding to the solutions (1.10) of (1.8), provided that ϕ can be chosen to satisfy the equation arising from the vanishing of the variation of the spin-1/2 fields. If so, then the background will preserve at least a fraction $m/2N$ of the supersymmetry.

The extra condition on a spinor satisfying (1.8) from the vanishing of the variation of the spin-1/2 fields is often of the form

$$\left[\Gamma^m\partial_m\phi - Y\right]\epsilon = 0 \tag{1.13}$$

for some $Y(\phi)$. Then the integrability conditions from (1.8), (1.13) are consistent with the field equations from a lagrangian of the form (1.2) only if

$$Y = 2(D-2)w' \tag{1.14}$$

and the potential takes the form (1.3). The examples that will be explored in later sections have a diagonal matrix

$$W_{ab} = w\delta_{ab} \tag{1.15}$$

and the backgrounds satisfying (1.6) preserve half the supersymmetry.

In section 2, truncations of certain supergravity models will be studied and found to have Killing spinor conditions which are precisely of the form (1.8), (1.9), so that there are BPS domain walls corresponding to solutions of (1.6). In particular, all of the non-compact gaugings of $N = 8$ supergravity, including those with potential of the form (1.1), and the $N = 4$ gauged supergravity of [5] will be shown to have BPS domain wall solutions. The superpotential in all these supergravity cases is of the form

$$w = c_1 e^{-a_1\phi} + c_2 e^{a_2\phi} \tag{1.16}$$

for some constants a_1, a_2, c_1, c_2.

Domain wall solutions of supergravities, and in particular of those with superpotentials of the form (1.16), have been extensively studied [13, 15, 16]. If $x_1 = 0$ or $x_2 = 0$, then the potential is an exponential, and the domain wall solutions were found in [17, 18, 19]. The case in which both x_1, x_2 are non-zero and $a_1 \neq a_2$ has been studied in [18, 13]. The case in which both x_1, x_2 are non-zero and $a_1 = a_2$ has received little attention, however, and this is the case that arises in some of the gauged supergravities with de Sitter solutions. In section 3, the domain wall solutions for potentials of the form (1.16) will be discussed. While this paper was in preparation, the paper [20] appeared which has some overlap with the results presented here.

2. Gauged supergravity

2.1 $N = 4$, $D = 4$ gauged supergravity

The ungauged $N = 4$ supergravity in $D = 4$ has a global $\mathrm{SU}(4) \times \mathrm{SL}(2,\mathbb{R})$ symmetry and a local $\mathrm{U}(4)$ symmetry. The $\mathrm{SU}(4) \times \mathrm{SL}(2,\mathbb{R})$ is a duality symmetry of the equations of motion and only a $\mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{SO}(1,1)$ subgroup is a symmetry of the action. The bosonic sector consists of 6 vector fields transforming as a $(\mathbf{6},\mathbf{1})$ of $\mathrm{SO}(4) \times \mathrm{SO}(1,1)$ and a complex scalar ϕ, taking values in the coset $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$. Gauging consists of promoting the rigid $\mathrm{SU}(2) \times \mathrm{SU}(2)$ symmetry to a local one with coupling constants g_1, g_2 for the two $\mathrm{SU}(2)$ factors, with the 6 vector fields becoming the gauge fields, and adding g_1, g_2-dependent terms, including a scalar potential, to

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obtain a supersymmetric theory. The scalar potential of [5] can be written in the form [21]

$$V = -\frac{1}{2}\left[(g_1^2 + g_2^2)\cosh(2|\phi|) + 4g_1g_2 + (g_1^2 - g_2^2)\frac{Re(\phi)}{|\phi|}\sinh(2|\phi|)\right]. \tag{2.1}$$

Field redefinitions bring the theory to one of three distinct cases with $\xi = g_2/g_1$ being $1, -1$ or 0 [22, 21]. First, if $g_1 = g_2$, one obtains the gauging of [1] with a potential

$$V = -\frac{1}{2}g^2(\cosh(2|\phi|) + 2) \tag{2.2}$$

and there is a supersymmetric AdS solution with

$$\Lambda = -\frac{3}{2}g^2 . \tag{2.3}$$

If $g_1 = -g_2$, one obtains the gauging of [5] with a θ-independent potential

$$V = -\frac{1}{2}g^2(\cosh(2|\phi|) - 2) \tag{2.4}$$

with a maximum at $\phi = 0$ so that there is a non-supersymmetric de Sitter solution with positive cosmological constant

$$\Lambda = \frac{1}{2}g^2 . \tag{2.5}$$

Finally, if $g_2 = 0$, one obtains the SU(2) gauging of [23]. These three theories can be obtained as consistent truncations of the $N = 8$ theories with gauge groups SO(8), SO(4, 4) and CSO(4, 4) respectively, to be reviewed in the next section.

Setting $\mathrm{Im}(\phi) = 0$, the dependence on $\varphi = \sqrt{2}Re(\phi)$ can be written as

$$V = -g^2\left(e^{\sqrt{2}\varphi} + 4\xi + \xi^2 e^{-\sqrt{2}\varphi}\right) \tag{2.6}$$

and this can be written as (1.3) with superpotential

$$w = \frac{1}{2\sqrt{2}}g\left(e^{\frac{1}{\sqrt{2}}\varphi} + \xi e^{-\frac{1}{\sqrt{2}}\varphi}\right) \tag{2.7}$$

and the Killing spinor conditions reduce to (1.8), (1.9) if the only non-zero matter field is φ.

2.2 $N = 8$, $D = 4$ gauged supergravity

The ungauged $N = 8$ supergravity in $D = 4$ has a global $E_{7(7)}$ symmetry and a local SU(8) symmetry [24]. The $E_{7(7)}$ is a duality symmetry of the equations of motion, but there is an SL$(8, \mathbb{R})$ subgroup which is a symmetry of the action. The bosonic sector consists of 28 vector fields transforming as a **28** of SL$(8, \mathbb{R})$ and 70 scalars, taking values in the coset $E_7/\,\mathrm{SU}(8)$. Gauging consists of promoting a 28-dimensional subgroup K of $\mathrm{SL}(8,\mathbb{R})$ to a local symmetry. The 28 vector fields become the gauge bosons, so that it is necessary that the subgroup K is chosen so that the **28** of $\mathrm{SL}(8,\mathbb{R})$ becomes the adjoint of K. Then supersymmetry requires the addition of terms depending on the coupling constant g to the action and supersymmetry transformation rules, including a scalar potential proportional to g^2. In [2], the gauging with $K = \mathrm{SO}(8)$ was constructed, and in [6]–[7] gaugings were constructed with non-compact gauge groups $K = \mathrm{SO}(p, 8-p)$ or the non-semi-simple gauge groups $\mathrm{CSO}(p,q,r)$ for all non-negative integers p,q,r with $p+q+r=8$. Here $\mathrm{CSO}(p,q,r)$ is the group contraction of $\mathrm{SO}(p+r,q)$ preserving a symmetric metric with p positive eigenvalues, q negative ones and r zero eigenvalues. Then $\mathrm{CSO}(p,q,0) = \mathrm{SO}(p,q)$ and $\mathrm{CSO}(p,q,1) = \mathrm{ISO}(p,q)$. The Lie algebra of $\mathrm{CSO}(p,q,r)$ is

$$[L_{ab}, L_{cd}] = L_{ad}\eta_{bc} - L_{ac}\eta_{bd} - L_{bd}\eta_{ac} + L_{bc}\eta_{ad}\,, \tag{2.8}$$

where

$$\eta_{ab} = \begin{pmatrix} \mathbf{1}_{p\times p} & 0 & 0 \\ 0 & \mathbf{1}_{q\times q} & 0 \\ 0 & 0 & 0_{r\times r} \end{pmatrix}. \tag{2.9}$$

$a,b = 1,\cdots,8$ and $L_{ab} = -L_{ba}$. Note that despite the non-compact gauge groups, these are unitary theories, as the vector kinetic term is not the minimal term constructed with the indefinite Cartan-Killing metric, but is constructed with a positive definite scalar-dependent matrix. The $\mathrm{CSO}(p,q,r)$ gauging and the $\mathrm{CSO}(q,p,r)$ gauging are equivalent. In [25], it was argued that these are the only possible gauge groups.

The 70 scalars consist of 35 scalars parameterising the coset $\mathrm{SL}(8,\mathbb{R})/\,\mathrm{SO}(8)$ and which can be represented by an 8×8 unimodular matrix S, and 35 pseudo-scalars parameterised by an anti-self-dual 4-form of $\mathrm{SL}(8,\mathbb{R})$. Let $K_{p,q,\xi}$ be the subgroup of $\mathrm{SL}(8,\mathbb{R})$ whose algebra is (2.8) with

$$\eta_{ab} = \begin{pmatrix} \mathbf{1}_{p\times p} & 0 \\ 0 & \xi\mathbf{1}_{q\times q} \end{pmatrix} \tag{2.10}$$

parameterised by ξ, with $p+q=8$. For $\xi = 1$, this is $\mathrm{SO}(8)$, for $\xi=-1$ this is $\mathrm{SO}(p,q)$ and for $\xi = 0$ this is the non-semi-simple $\mathrm{CSO}(p,0,q)$, which it is convenient to abbreviate to $\mathrm{CSO}(p,q)$. For other $\xi > 0$ ($\xi < 0$), this is isomorphic to the algebra with $\xi = 1$ ($\xi = -1$).

In [7], the potential $V(s)$ of the $N=8$ theory with gauge group $K_{p,q,\xi}$ ($p+q=8$) was found explicitly for the case in which there is only one non-zero scalar s, which is the $\mathrm{SO}(p)\times\mathrm{SO}(8-p)$ invariant scalar represented by the $\mathrm{SL}(8,\mathbb{R})$ matrix

$$S = \begin{pmatrix} e^{-s}\mathbf{1}_{p\times p} & 0 \\ 0 & e^{ps/q}\mathbf{1}_{q\times q} \end{pmatrix}. \tag{2.11}$$

The potential is [7]

$$V_{p,q,\xi} = -\frac{1}{32} g^2 e^{2s} \left(2p(q-3p) - 16pq\xi e^{-8s/q} + 2q(p-3q)\xi^2 e^{-16s/q} \right) . \tag{2.12}$$

Explicitly, this gives [7]

$$\begin{aligned}
V_{7,1,\xi} &= \frac{1}{8} g^2 \left(-35 e^{2s} - 14\xi e^{-6s} + \xi^2 e^{-14s} \right) , \\
V_{6,2,\xi} &= -3g^2 \left(e^{2s} + \xi e^{-2s} \right) , \\
V_{5,3,\xi} &= -\frac{3}{8} g^2 \left(5e^{2s} + 10\xi e^{-2s/3} + \xi^2 e^{-10s/3} \right) , \\
V_{4,4,\xi} &= -g^2 \left(e^{2s} + 4\xi + \xi^2 e^{-2s} \right) , \\
V_{3,5,\xi} &= -\frac{3}{8} g^2 \left(e^{2s} + 10\xi e^{2s/5} + 5\xi^2 e^{-6s/5} \right) , \\
V_{2,6,\xi} &= -3g^2 \xi \left(e^{2s/3} + \xi e^{-2s/3} \right) , \\
V_{1,7,\xi} &= \frac{1}{8} g^2 \left(e^{2s} - 14\xi e^{6s/7} - 35\xi^2 e^{-2s/7} \right) .
\end{aligned} \tag{2.13}$$

From the argument of [12], any critical point of these potentials $V(s)$ is also a critical point of the full potential. The SO(8) gauging has a critical point of $V_{p,q,1}$ at $s=0$ preserving SO(8) and all 32 supersymmetries. The other critical points of these potentials break all supersymmetries and are as follows [7]. The potential $V_{7,1,1}$ has two critical points and so the SO(8) gauging has two SO(7)-invariant critical points with $\Lambda < 0$, the one at $s=0$, and one which breaks all supersymmetries. The potential for SO(4, 4) is of the form (1.1) and has a $\Lambda > 0$ critical point at $s=0$. The SO(5, 3) gauging (which is equivalent to the SO(3, 5) gauging) also has a $\Lambda > 0$ critical point, at $s = -\frac{3}{8}\log 3$, and the potential is of the form (1.16) with $a_1 \neq a_2$. When $\xi = 0$, these potentials are all of the form

$$V \propto -e^{2s} \tag{2.14}$$

and in the case $p=2$, the potential $V_{2,6,0}$ vanishes identically, and there is a $\Lambda = 0$ critical point of the full potential, so that there is a non-supersymmetric Minkowski space solution.

The scalar field s can be written in terms of a canonically normalised scalar ϕ by

$$\phi = \sqrt{\frac{2p}{q}} s . \tag{2.15}$$

In the Killing spinor condition (1.12), the tensor W^{ab} is given by the so-called A_1^{ab} tensor of [6, 7], and for a background of the $K_{p,q,\xi}$ gauging with ϕ the only non-vanishing matter field, this is of the form (1.15) with [7]

$$w_{p,q,\xi}(\phi) = \frac{\sqrt{2}g}{8} \left(p e^{\sqrt{\frac{q}{2p}}\phi} + q\xi e^{-\sqrt{\frac{p}{2q}}\phi} \right) . \tag{2.16}$$

Then the Killing spinor conditions are precisely (1.8), (1.9) with w given by (2.16) and it is straightforward to check that the potential (2.12) is given in terms of this superpotential $w_{p,q,\xi}$ by (1.3) with $D = 4$.

2.3 $N = 8$ gauged supergravity in $D = 5$ and $D = 7$

The ungauged $N = 8$ supergravity in $D = 5$ has an action invariant under a global $E_{6(6)}$ symmetry and a local USp(8) symmetry [26]. The bosonic sector consists of 27 vector fields transforming as a **27** of E_6 and 42 scalars, taking values in the coset $E_6/\mathrm{USp}(8)$. It has a dual form with only $\mathrm{SL}(6,\mathbb{R}) \times \mathrm{SL}(2,\mathbb{R})$ global symmetry in which 12 of the vector fields are dualised to 2-forms, leaving 15 vector fields transforming as the (**15**,**1**) of $\mathrm{SL}(6,\mathbb{R}) \times \mathrm{SL}(2,\mathbb{R})$. The gaugings arise from making a 15-parameter subgroup K of $\mathrm{SL}(6,\mathbb{R})$ local, with the 15 vectors becoming the gauge bosons. The gaugings with $K = \mathrm{SO}(6)$ [4, 3] and $K = \mathrm{SO}(p, 6-p)$ [3] arise in this way, but non-semi-simple gaugings arise from a slightly different construction [27] and will not be discussed here.

Twenty of the 42 scalars parameterise the coset $\mathrm{SL}(6,\mathbb{R})/\mathrm{SO}(6)$ and can be represented by a 6×6 unimodular matrix S. For the SO(6) or $\mathrm{SO}(p,q)$ gauging with $p + q = 6$, consider the $\mathrm{SO}(p) \times \mathrm{SO}(6-p)$ invariant scalar $\phi(x)$ represented by the $\mathrm{SL}(6,\mathbb{R})$ matrix

$$S = \begin{pmatrix} e^{-a\sqrt{\frac{p}{q}}\phi}\mathbf{1}_{p\times p} & 0 \\ 0 & e^{a\sqrt{\frac{q}{p}}\phi}\mathbf{1}_{q\times q} \end{pmatrix} \tag{2.17}$$

as in [3], where a is a normalisation. Then for a scalar background with $\phi(x)$ the only-non-vanishing matter field, the Killing spinor conditions for a background of the SO(6) ($\xi = 1$) or $\mathrm{SO}(p,q)$ ($\xi = -1$) gauging are precisely (1.12), (1.15) with w proportional to $tr(\eta S)$ where η is the $\mathrm{SO}(p,q)$ invariant metric [3] so that

$$w_{p,q,\xi}(\phi) \propto g\left(pe^{a\sqrt{\frac{q}{p}}\phi} + q\xi e^{-a\sqrt{\frac{p}{q}}\phi}\right). \tag{2.18}$$

The ungauged $N = 8$ supergravity in $D = 7$ has a rigid $\mathrm{SL}(5,\mathbb{R})$ symmetry and a local SO(5) symmetry, with scalars taking values in $\mathrm{SL}(5,\mathbb{R})/\mathrm{SO}(5)$. In [28], it was shown that one can gauge an $\mathrm{SO}(p,q)$ subgroup of the $\mathrm{SL}(5,\mathbb{R})$ symmetry for any $p+q = 5$. For the SO(5) or $\mathrm{SO}(p,q)$ gauging with $p+q = 5$, the $\mathrm{SO}(p) \times \mathrm{SO}(5-p)$ invariant scalar $\phi(x)$ can be represented by the $\mathrm{SL}(5,\mathbb{R})$ matrix (2.17). The Killing spinor conditions for such a background of the SO(5) ($\xi = 1$) or $\mathrm{SO}(p,q)$ ($\xi = -1$) gauging are again (1.12), (1.15) with w again of the form (2.18).

3. Solutions

3.1 The ansatz

The metric (1.4) can be brought to the form

$$ds^2 = e^{2A(r)}ds^2\left(\mathbb{E}^{(1,D-2)}\right) + e^{2B}dr^2 \tag{3.1}$$

for any function $B(r)$ by a coordinate transformation $r \to f(r)$ with $f' = e^{-B}$. The equations (1.6) then become

$$\begin{aligned} \partial_r A &= \mp 2w(\phi)e^B \\ \partial_r \phi &= \pm 2(D-2)w'(\phi)e^B \,. \end{aligned} \tag{3.2}$$

A useful choice is

$$B = \lambda A \tag{3.3}$$

for some constant λ, as this simplifies the equations in some cases. Choosing basis 1-forms

$$e^\mu = e^A dx^\mu, \qquad e^r = e^{\lambda A} dr \tag{3.4}$$

the curvature 2-form Θ^{MN} has the frame components

$$\begin{aligned} \Theta^{r\mu} &= (-A'' + (\lambda-1)(A')^2)e^{-2\lambda A} e^r \wedge e^\mu \,, \\ \Theta^{\mu\nu} &= -(A')^2 e^{-2\lambda A} e^\mu \wedge e^\nu \,. \end{aligned} \tag{3.5}$$

Consider the case in which the superpotential is of the form

$$w = c_1 e^{-a_1\phi} + c_2 e^{a_2\phi} \tag{3.6}$$

for some constants a_1, a_2, c_1, c_2, with $a_1, a_2 > 0$. All of the supergravity theories discussed in the last section have superpotentials of this form. There are a number of different cases, depending on whether or not $a_1 = a_2$, and these will be considered separately.

3.2 Generic case: $a_1 \neq a_2$

A solution of the equations (3.2) was found in [18]. It is convenient to choose $B = \lambda A$ with

$$\lambda = \frac{a_1 a_2}{(D-2)(a_2 - a_1)} \,. \tag{3.7}$$

Then (3.2) are solved by

$$e^{(a_1 - a_2)\phi} = r \tag{3.8}$$

and

$$e^{-\lambda A} = x_1\, r^{\frac{a_2}{a_2 - a_1}} + x_2\, r^{-\frac{a_1}{a_2 - a_1}} \,, \tag{3.9}$$

where

$$x_1 = \mp 2(D-2)a_1 c_1, \qquad x_2 = \pm 2(D-2)a_2 c_2 \,. \tag{3.10}$$

If $c_1 = 0$ or $c_2 = 0$, the potential is a simple exponential and the domain wall solutions of [17, 18, 19] are recovered. In this case, either the region as $r \to \infty$ or the region as $r \to 0$ is asymptotically flat, but the scalar ϕ is proportional to $\log r$. The regions $r \to 0$ and $r \to \infty$ can be interchanged by the coordinate transformation $r \to 1/r$, so that the asymptotically flat region can always be arranged to be at large r. With the asymptotically flat region arranged to be at large r, writing $r = |z|$ gives a solution defined for all real z that is asymptotically flat as $z \to \pm\infty$ and is symmetric under the reflection $z \to -z$ and has a singular domain wall at $z = 0$.

If both c_1, c_2 are non-zero and have opposite sign, the frame components of the curvature diverge both as $r \to 0$ and as $r \to \infty$ [18]. Such models arise from the $\mathrm{SO}(p,q)$ gauged supergravities in $D = 4, 5$ with $p \neq q$ and $\xi = -1$

If c_1, c_2 are non-zero and have the same sign, then $e^{-\lambda A}$ vanishes at $r = r_c$ where

$$r_c^{\frac{a_2+a_1}{a_2-a_1}} = \frac{a_2 c_2}{a_1 c_1} \tag{3.11}$$

and it is necessary to restrict to the region in which $e^{-\lambda A}$ is positive. Suppose the signs are such that the solution is restricted to the region $0 \leq r \leq r_c$. The solution is singular at $r = 0$, but the curvature vanishes at $r = r_c$ and the scalar field is finite there, so there is a flat region near $r = r_c$, which is at an infinite distance from any point with $r < r_c$. Again, writing $r = |z|$ gives a solution defined for real $|z| < r_c$ which is asymptotically flat as $z \to \pm r_c$ and which has a singular domain wall at $z = 0$. Such models arise from the $N = 8$ theories with compact gauge groups, with ϕ the $\mathrm{SO}(p) \times \mathrm{SO}(q)$ invariant scalar, with $p \neq q$.

3.3 Special case: $a_1 = a_2$ and $c_1 = -c_2$

In this case

$$w = c \sinh(a\phi)\,. \tag{3.12}$$

Taking $\lambda = 0$ and setting $\varphi = a\phi$, (1.6) gives

$$\partial_r \varphi = \alpha \cosh \varphi\,, \tag{3.13}$$

where

$$\alpha = \pm 2(D-2)ca\,. \tag{3.14}$$

This can be integrated to give

$$\alpha(r - r_0) = gd(\varphi)\,, \tag{3.15}$$

where $gd(\varphi)$ is the *gudermannian* defined by

$$gd(x) = \int_0^x \frac{dt}{\cosh t} \tag{3.16}$$

and can be written as

$$gd(x) = 2 \arctan(e^x) - \frac{\pi}{2}\,. \tag{3.17}$$

It enjoys many remarkable properties, such as

$$\begin{aligned} \tanh(x) &= \sin(y) \\ \sinh(x) &= \tan(y) \\ \cosh(x) &= \sec(y)\,, \end{aligned} \tag{3.18}$$

where $y = gd(x)$. Then (3.15),(3.18) imply

$$\sinh(\varphi) = \tan[\alpha(r - r_0)] \tag{3.19}$$

or, after a shift of r to absorb constants of integration,

$$e^{\varphi} = \tan(\alpha r'/2) \tag{3.20}$$

with $r' = r - r_0 + \pi/(2\alpha)$. Then A satisfies

$$\partial_r A = \mp 2c \sinh\varphi = \mp 2c \tan[\alpha(r - r_0)] \tag{3.21}$$

and so

$$e^{A} = (\cos[\alpha(r - r_0)])^{\pm 2c/\alpha} \; . \tag{3.22}$$

Note that in this solution r is restricted to the region

$$|\alpha(r - r_0)| < \frac{\pi}{2} \tag{3.23}$$

as at $|\alpha(r - r_0)| = \pi/2$ the scalar field φ becomes infinite and e^A either vanishes or diverges, depending on the choice of sign in (1.6). However, for either sign

$$\Theta^{\mu\nu} = -4c^2 \tan^2(\alpha(r - r_0)) e^{\mu} \wedge e^{\nu} \tag{3.24}$$

so that the frame components of the curvature diverge at the boundaries $|\alpha(r - r_0)| = \pi/2$.

3.4 Special case: $a_1 = a_2$ and $c_1 = c_2$

In this case

$$w = c\cosh(a\phi) \, . \tag{3.25}$$

Taking $\lambda = 0$ and setting $\varphi = a\phi$, (1.6) gives

$$\partial_r \varphi = \alpha \sinh\varphi \, , \tag{3.26}$$

where

$$\alpha = \pm 2(D - 2)ca \, . \tag{3.27}$$

This can be integrated to give

$$e^{\alpha(r - r_0)} = \tanh(\varphi/2) = \sqrt{\frac{\cosh\varphi - 1}{\cosh\varphi + 1}} \tag{3.28}$$

so that

$$\cosh\varphi = f[\alpha(r - r_0)] \, , \tag{3.29}$$

where

$$f(y) = \frac{1 + e^{2y}}{1 - e^{2y}}\,. \tag{3.30}$$

Then A satisfies

$$\partial_r A = \mp 2c \cosh \varphi = \mp 2c f[\alpha(r - r_0)] \tag{3.31}$$

which implies

$$A = \mp 2c \left((r - r_0) - \alpha^{-1} \log \left[1 - e^{2\alpha(r - r_0)} \right] \right). \tag{3.32}$$

There is a singularity at $r = r_0$ where φ, A and the curvature 2-form diverge. If $\alpha > 0$, the solution is restricted to the region $r < r_0$ and if $\alpha < 0$, it is restricted to the region $r > r_0$. The solution is non-singular as $r \to \infty$ ($\alpha < 0$) or $r \to -\infty$ ($\alpha > 0$).

3.5 Special case: $a_1 = a_2 = a$ and $c_1 \neq c_2$

Here

$$w = c_1 e^{-a\phi} + c_2 e^{a\phi} \tag{3.33}$$

and there are two cases, depending on the relative signs of c_1, c_2. If $c_1 c_2 < 0$, then the solution is

$$\begin{aligned} e^{a\phi} &= \sqrt{\frac{-c_1}{c_2}} \tan \rho \\ e^{-A} &= \frac{1}{2} \sin(2\rho)\,, \end{aligned} \tag{3.34}$$

where

$$\rho = 2(D - 2) a \sqrt{-c_1 c_2} (r - r_0)\,. \tag{3.35}$$

If $c_1 c_2 > 0$, then the solution is

$$\begin{aligned} e^{a\phi} &= \sqrt{\frac{c_1}{c_2} \frac{e^\rho - 1}{e^\rho + 1}} \\ A &= \pm \frac{2}{(D - 2) a \sqrt{c_1 c_2}} \left[\rho - \log \left(e^{2\rho} - 1 \right) \right], \end{aligned} \tag{3.36}$$

where

$$\rho = 2(D - 2) a \sqrt{c_1 c_2} (r - r_0) \tag{3.37}$$

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